NASA TECHNICAL TRANSLATION NASA TT-20110

IN-39

97312

THERMOPLASTICITY OF COUPLED BODIES IN THE CASE OF STRESS-DEPENDENT HEAT TRANSFER

17P

O. A. Kilikovskaya

Translation of O termouprugosti sopryazhennykh tel pri teplootdache, zavisyashchey ot zapryazheniy, IN: Uprugost' i Neuprugost', no. 3, 1973, pp. 47-60, 73A39327.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 August 1987

(NASA-TT-20110) THERMOPLASTICITY OF COUPLED BODIES IN THE CASE OF STRESS-DEPENDENT HEAT TRANSFER (NASA) 17 p Avail: NTIS HC A02/MF A01 CSCL 20K G5/39

N87-28070

Unclas 0097312

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
TT-20110		

4. Title and Subtitle	5. Report Date
THERMOPLASTICITY OF COUPLED BODIES IN THE CASE OF STRESS-DEPENDENCE HEAT TRANSFER	August 1987
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
O. A. Kilikovskaya	10. Work Unit No.
	11. Contract or Grant No. NASw- 4004
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108	13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Translation of O termouprugosti sopryazhennykh tel pri teplootdache, zavisyashchey ot zapryazheniy, IN: Uprugost' i Neuprugost', no. 3, 1973, pp. 47-60, 73A39327.

16. Abstract The problem of temperature stresses in solid, deformable bodies, on whose surfaces there is heat transfer under conditions of "non-ideal contact", was discussed by Boll and Winer [1]. This article gives the formulation of the "connected" problem of this type, i.e., it is assumed that the heat transfer coefficient at the boundary of two bodies depends on the stress state in the bodies, for example, it is a function of the normal pressure at the boundary of the two bodies. The solutions of certain one-dimensional problems are given in this formulation. The conditions for the existence and uniqueness in general form are not discussed in this article.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified and Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	14	

THERMOPLASTICITY OF COUPLED BODIES IN THE CASE OF STRESS-DEPENDENT HEAT TRANSFER

*/47

O. A. Kilikovskaya

1. Formulation of the problem. Let us formulate this nonlinear problem for the case of two bodies.

There are two bodies occupying the volume V_1, V_2 with the surraces Σ_1, Σ_2. At points P belonging to the overall section of the boundary $\Sigma_{1,2}$, the conditions for coupling of displacements and stresses are satisfied

$$\bar{\sigma}_n^{(1)}(P, t) = \bar{\sigma}_n^{(2)}(P, t), \tag{1.1}$$

$$\bar{u}^{(1)}(P, t) = \bar{u}^{(2)}(P, t) + \bar{f}(\sigma). \tag{1.2}$$

In the overall section of the boundary $\Sigma_{1,2}$, we can give the heat transfer conditions in the form

$$k_1 \frac{\partial T_1}{\partial n} = k_2 \frac{\partial T_2}{\partial n}, \tag{1.3}$$

$$k_1 \frac{\partial T_1}{\partial n} = \frac{1}{R(\sigma_n)}(T_2 - T_1). \tag{1.4}$$

Here T_i--temperature; k_i--heat conductivity of contacting bodies coefficient; n--external normal to the contact surface $\Sigma_{1,2}$ with respect to the body 1, $R(\sigma_n)$--so-called contact resistance [1], which we shall assume is a function of the normal pressure on the surface. Equation (1.2) just like equation (1.4), considers the "non-ideal contact" of the bodies. On the remaining sections of the surfaces Σ_1 and Σ_2, which do not have common points, the classical boundary conditions of thermoelasticity are satisfied.

2. One-dimensional problems assuming an exact solution. We shall assume that in (1.2) $f(\sigma) \equiv 0$ for purposes of simplicity.

---

*Numbers in margin indicate pagination of foreign text.

1. Nonstationary thermoelasticity in a layer. The layer $x_0 \leqslant x \leqslant x_1$ at the temperature T_o moves between two rigid plates with the temperature T_1 and T_2. In this case the problem is reduced to solving the equation

$$\frac{\partial \sigma_{xx}}{\partial x} = 0, \tag{2.1}$$

/48

$$\sigma_{xx} = \widetilde{E}\left(\frac{\partial u_x}{\partial x} + \widetilde{\alpha}(T - T_0)\right), \tag{2.2}$$

$$k\frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t}. \tag{2.3}$$

Under the initial conditions

$$u(x)|_{t=0} = 0, \tag{2.4}$$

$$T(x)|_{t=0} = T_0. \tag{2.5}$$

Under the boundary conditions

$$\text{at } x = x_0 \quad u(t) = 0, \tag{2.6}$$

$$k\frac{\partial T}{\partial x} = \frac{1}{R(\sigma)}(T(x_0) - T_1), \tag{2.7}$$

$$\text{at } x = x_1 \quad u(t) = 0, \tag{2.8}$$

$$k\frac{\partial T}{\partial x} = \frac{1}{R(\sigma)}(T_2 - T(x_1)). \tag{2.9}$$

Here for the case of one-dimensional stress state $\widetilde{E} = E$ --elasticity modulus; $\widetilde{\alpha} = \alpha$ --linear expansion coefficient, where

$$\widetilde{E} = E\frac{(1-\nu)}{(1+\nu)(1-2\nu)}, \quad \widetilde{\alpha} = \alpha\frac{1+\nu}{1-\nu},$$

where ν --Poisson coefficient.

Integrating equations (2.1) and (2.2), considering the boundary conditions (2.6), (2.8), we obtain

$$\sigma_{xx}(x, t) = \frac{-\widetilde{E}\widetilde{\alpha}}{x_1 - x_0}\int_{x_0}^{x_1}(T(x, t) - T_0)\,dx \equiv \sigma^*(t). \tag{2.10}$$

Integrating equation (2.3) with respect to x from x_o to x_1, we obtain

$$k\frac{\partial T}{\partial x}\Big|_{x_1} - k\frac{\partial T}{\partial x}\Big|_{x_0} = \rho c \frac{\partial}{\partial t}\int_{x_0}^{x_1} (T(x, t - T_0)\, dx.$$

Using the boundary conditions (2.7) and (2.9) and equation (2.10), we obtain

$$\frac{1}{R(\sigma^*)}[T_2 + T_1 - T(x_1, t) - T(x_0, t)] = -\rho c \frac{x_1 - x_0}{\widetilde{\alpha}\widetilde{E}} \cdot \frac{d\sigma^*}{dt}. \quad (2.11)$$

/49

Integrating (2.11), we obtain

$$\sigma^*(t) = \Phi^{-1}\left(\frac{1}{\gamma}\int_0^t (T(x_1) + T(x_0) - T_1 - T_2)\, dt\right), \quad (2.12)$$

where

$$\Phi(\sigma^*) = \int_0^{\sigma^*} R(\sigma)\, d\sigma,$$

$$\gamma = \rho c \frac{x_1 - x_0}{\widetilde{\alpha}\widetilde{E}}.$$

From the connected system (2.1)-(2.3) with the boundary conditions (2.6)-(2.9), the equation of heat conductivity with the non-linear conditions is derived

$$k\frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t},$$

$$k\frac{\partial T}{\partial x}\Big|_{x_0} = \frac{T(x_0) - T_1}{R\left\{\Phi^{-1}\left[\frac{1}{\gamma}\int_0^t (T(x_1) + T(x_0) - T_1 - T_2)\, dt\right]\right\}},$$

$$k\frac{\partial T}{\partial x}\Big|_{x_1} = \frac{T_2 - T(x_1)}{R\left(\Phi^{-1}\left(\frac{1}{\gamma}\int_0^t (T(x_1) + T(x_0) - T_1 - T_2)\, dt\right)\right)}.$$

We shall designate solution of this problem of heat conductivity by T. Then we find the stress according to (2.12).

2. Thermal stress in the n^{th} plate. Let us examine a system of n layers $x_0 \leqslant x \leqslant x_1, \ldots, x_{n-1} \leqslant x \leqslant x_n$. The characteristics

of the i^{th} layer are designated by E_i, α_i, $\rho_i c_i$, K_i. On the surfaces of the first and the n^{th} layers the fluxes $q_o(t)$ and $q_n(t)$ exist. Let us consider a certain steady mode in which a quantity of heat Q is introduced through a unit of surface, and the steady flux q^∞ exists. The displacements $u(x_o)$, $u(x_n)$ are given. The system of equations has the form

$$\frac{\partial \sigma_{xx}^{(i)}}{\partial x} = 0, \tag{2.13}$$

$$K_i \frac{\partial^2 T}{\partial x^2} = 0, \tag{2.14}$$

$$\sigma_i = E_i\left(\frac{\partial u}{\partial x} - \alpha_i T\right). \tag{2.15}$$

At the boundary of the i^{th} and the $i+1^{th}$ layers, the following conditions are satisfied /50

$$K_1 \frac{\partial T}{\partial x}\Big|_{x_0} = q^\infty, \tag{2.16}$$

$$K_{i+1} \frac{\partial T_{i+1}}{\partial x}\Big|_{x_i} = K_i \frac{\partial T_i}{\partial x}\Big|_{x_i}, \tag{2.17}$$

$$R_i^{i+1}(\sigma) K_i \frac{\partial T_i}{\partial x}\Big|_{x_i} = (T_{i+1} - T_i)|_{x_i}, \tag{2.18}$$

$$u(x_0) = u_0, \quad u(x_n) = u_n,$$

$$u^{i+1}(x_i) = u^i(x_i),$$

$$\sigma_{xx}^{i+1}|_{x_i} = \sigma_{xx}^i|_{x_i}. \tag{2.19}$$

The solution of (2.14) with the boundary conditions (2.16)-(2.18) is determined (even at $R_i^{i+1}(\sigma) = \text{const}$) within an accuracy of a factor which is found as follows.

Let us examine the solution of the heat conductivity problem until it reaches the steady mode

$$K_i \frac{\partial^2 T_i}{\partial x^2} = \rho_i c_i \frac{\partial T_i}{\partial t},$$

Integrating each equation with respect to x from x_{i-1} up to x_i, and with respect to t from zero to t, adding and subtracting the boundary condition (2.17), we obtain

$$\int_0^t (q_0(t) - q_n(t))\,dt = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \rho_i c_i T_i(x, t)\,dx \equiv Q. \quad (2.20)$$

Beginning with a certain time t_1, we assume $q_o = q_n = q^\infty$. The integral in the left side of (2.20) which does not depend on $R_i^{i+1}(\sigma)$, represents the amount of heat introduced into the system. We shall designate it by Q. Thus, the system of equations and the conditions (2.13)-(2.20) is closed and describes the thermoelastic state for $t \gg t_1$. The solution of (2.13), (2.15) with the boundary conditions (2.19) gives

$$\sigma_{xx}^i(x) \equiv \sigma = \frac{1}{\sum_{i=1}^{n} \frac{h_i}{E_i}} \left[u_n - u_0 - \sum_{i=1}^{n} \alpha_i \int_{x_{i-1}}^{x_i} T_i(x)\,dx \right]. \quad (2.21)$$

The solution of the equations at $R_i^{i+1} = 0$, i.e., by replacing the boundary condition (2.18) by the condition of ideal contact

$$T_{i+1}|_{x_i} = T_i|_{x_i}, \quad (2.22)$$

we shall call $T^{\text{ид}}$, $\sigma^{\text{ид}}$.

/51

The solution of (2.14) with the boundary conditions (2.16), (2.17), (2.20) and the condition of ideal contact (2.22) is represented by a solid line with the angular coefficients $\frac{\partial T_i^{\text{ид}}}{\partial x} = -\frac{q^\infty}{k_i}$ at $x_{i-1} \leqslant x \leqslant x_i$, and with a certain ordinate T_n at $x = x_n$, which may be readily determined from (2.20) (Figure 1).

The solution of equation (2.14) with the condition of non-ideal contact (2.18) has the form

$$T_i(x) = T_i^{\text{ид}} + \Delta T_n + q^\infty \sum_{j=i}^{n-1} R_j^{j+1}(\sigma), \quad i = 1, \ldots, n-1,$$
$$T_n(x) = T_n^{\text{ид}}(x) + \Delta T_n, \quad (2.23)$$

where the parameter ΔT_n is the temperature decrease at the end point $x = x_n$ as compared with $T_n^{\text{ид}}$. Substituting (2.23) into (2.20) and (2.21), we obtain a system of equations for determining the constants σ and ΔT_n:

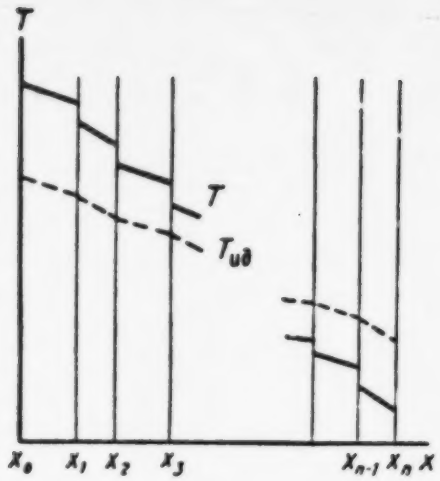

Figure 1. Temperature distribution in the n^{th} plate

$$Q = Q + \Delta T_n \sum_{i=1}^{n} \rho_i c_i h_i + q^{\infty} \sum_{i=1}^{n-1} \rho_i c_i h_i \sum_{j=i}^{n-1} R_j^{j+1}(\sigma),$$

$$(\sigma - \sigma^{ид}) \sum_{i=1}^{n} \frac{h_i}{E_i} = -\Delta T_n \sum_{i=1}^{n} \alpha_i h_i - q^{\infty} \sum_{i=1}^{n-1} \alpha_i h_i \sum_{j=i}^{n-1} R_j^{j+1}(\sigma), \qquad (2.24)$$

and excluding ΔT_n, we obtain the equation for determining σ:

$$\sigma = \sigma_{ид} - \sum_{j=1}^{n-1} R_j^{j+1}(\sigma)\, C_j, \qquad (2.25)$$

/52

where

$$C_j = \frac{q^{\infty}\left(\sum_{i=1}^{j} \alpha_i h_i \sum_{i=j+1}^{n} \rho_i c_i h_i - \sum_{i=1}^{j} \rho_i c_i h_i \sum_{i=j+1}^{n} \alpha_i h_i\right)}{\sum_{i=1}^{n} \frac{h_i}{E_i} \sum_{i=1}^{n} \rho_i c_i h_i}. \qquad (2.26)$$

An analysis of (2.25) and (2.26) shows that there may be three cases:

$$1)\ \sigma < \sigma_{ид};\ 2)\ \sigma > \sigma_{ид};\ 3)\ \sigma = \sigma_{ид}.$$

If $\frac{\alpha_i}{\rho_i c_i} = \text{const}$, which holds for layers of the same material, there is a third case $\sigma = \sigma_{ид}$. If α_i monotonically decreases with an increase in the number, and $\rho_i c_i$ does not decrease, then

$$\sum_{i=1}^{j} \alpha_i h_i \sum_{i=j+1}^{n} \rho_i c_i h_i - \sum_{i=1}^{j} \rho_i c_i h_i \sum_{i=j+1}^{n} \alpha_i h_i >$$

$$> \sum_{i=1}^{j} h_i \sum_{i=j+1}^{n} h_i (\alpha_j \rho_{j+1} c_{j+1} - \alpha_{j+1} \rho_j c_j) > 0.$$

Consequently, all $C_j > 0$ and $\sigma < \sigma_{нд}$.

If $\alpha_i < \alpha_{i+1}$ and $\rho c_i \geqslant \rho c_{i+1}$, then $c_i < 0$ and $\sigma > \sigma_{нд}$.

We shall study in greater detail the case of two layers. Equation (2.25) has the form

$$CR(\sigma) = \sigma_{нд} - \sigma, \tag{2.27}$$

where

$$\sigma_{нд} = -LQ - Fq, \quad C = Dq.$$

Here L, F, D are constants which depend only on geometric and physical characteristics of the layers

$$L = \frac{\sum_{1}^{2} \alpha_i h_i}{d^2}, \tag{2.28}$$

$$D = \frac{h_1 h_2}{d^2} (\rho_2 c_2 \alpha_1 - \rho_1 c_1 \alpha_2),$$

$$F = \frac{1}{2} D \sum_{1}^{2} \frac{h_i}{k_i},$$

where /53

$$d^2 = \sum_{1}^{2} \frac{h_i}{E_i} \sum_{1}^{2} \rho_i c_i h_i.$$

It can be seen from (2.28) that L does not depend on the location of the layers, and D and F change sign when the layers are moved with respect to the direction of the thermal flux. Thus, the temperature at a cold point of the system

$$T_2 = T_2^{\text{нд}} + \Delta T_2 = \frac{Q - q\left(\sum_1^2 \frac{1}{2}\rho_i c_i h_i^2/k_i + \rho_1 c_1 h_1 h_2/k_2\right)}{\sum_1^2 \rho_i c_i h_i} -$$

$$- R(\sigma)\, q^{\infty} \frac{\rho_1 c_1 h_1}{\sum_1^2 \rho_i c_i h_i}.$$

An increase or a decrease in σ as compared with 3b depends on the coefficient C=Dq. This can readily be explained physically. The temperature distribution in the case of a non-ideal thermal contact has the form $T_1 = T_1^{\text{нд}}(x) + \Delta T_1$,

$$T_2 = T_2^{\text{нд}}(x) - |\Delta T_2|.$$

Since $Q = Q_{\text{нд}}$, then $\Delta Q_1 = +|\Delta Q_2|$, i.e.,

$$\Delta T_1 \rho_1 c_1 h_1 = |\Delta T_2| \rho_2 c_2 h_2. \qquad (2.29)$$

Considering (2.29), the additional temperature expansion of the first layer and the additional contraction of the second layer are connected by the relationship

$$\frac{\Delta u_1}{|\Delta u_2|} = \frac{\alpha_1 \Delta T_1 h_1}{\alpha_2 |\Delta T_2| h_2} = \frac{\alpha_1 \rho_2 c_2}{\alpha_2 \rho_1 c_1}.$$

If $\alpha_1\rho_2c_2 > \alpha_2\rho_1c_1$, , then the additional temperature expansion of the first layer will be greater than the additional contraction of the second layer, and consequently the compressive stress with respect to the modulus must increase in the case of the same displacements of the boundaries x_o and x_n. In this case D>0 and equation (2.27) give the same result.

/54

The positive function R(σ) is determined at $\sigma \leqslant 0$ and increases. The solution of equation (2.27), if D>0 always exists and represents the abscissa of the point of intersection $f_1(\sigma) = CR(\sigma)$ and $f_2(\sigma) = \sigma_{\text{нд}} - \sigma$ (Figure 2).

If $\alpha_1\rho_2c_2 < \alpha_2\rho_1c_1$, , then D<0 and C<0. We now assume that $|DqR'(\sigma)| < 1$ for all σ<0. Then if $|CR(0)| < \sigma_{\text{нд}}$, the solution

exists and is unique, and at $|CR(0)|>\sigma_{ид}$ there is no solution in the negative region σ. Determining R in the region σ>0 and assuming that the problem does not lose any meaning for σ>0 (the layers are combined), we find that the solution also exists always (Figure 3). If in a certain region $|DqR'(\sigma)|>1$, then for certain very small Q several states are possible (Figure 4). In this case an analysis of the limiting state does not make it possible to select the solution. It is necessary to examine the history of the process.

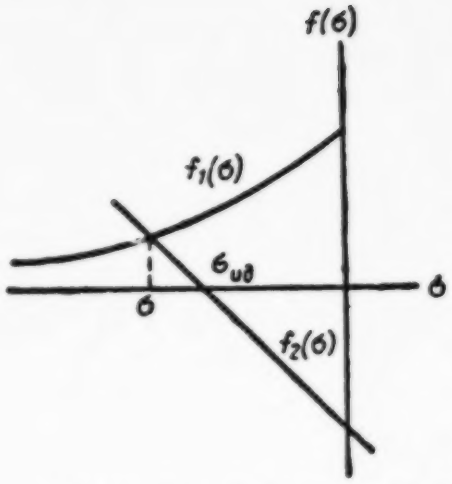

Figure 2. Solution of the equation $CR(\sigma)=\sigma_{ид}-\sigma$ at C>0

/55

For real, rather small R(σ), all of the special cases (the absence of a solution or the presence of several solutions) exist when $\sigma_{ид}$ are small for very large q^{∞}, and for a small amount of heat Q introduced. Then the non-ideal nature of the thermal contact causes the temperature to decrease in the second case to negative temperatures, and tensile stress may arise based on the condition of the problem.

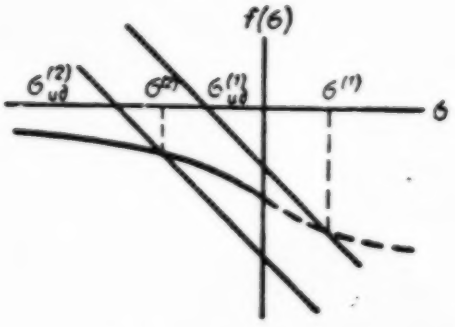

Figure 3. Solution of the equation $CR(\sigma)=\sigma_{ид}-\sigma$ at C<0 and $|CR'(\sigma)|<1$

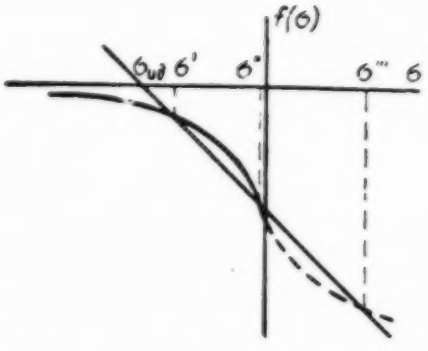

Figure 4. Solution of the equation at $CR(\sigma)=\sigma_{ид}-\sigma$ at C>0 and $|CR'(\sigma)|>1$ in a certain region σ

Let us consider the stress state of the n^{th} layer of the plate, when the steady temperature mode is given by the flux q^{∞} and the temperature at the end point $T(x_n)=T_n$.
The heat transfer coefficient with the external boundary and the temperature of the medium is reduced to the same condition: $T_n=T_{cp}+q^{\infty}\lambda$). Then the equation for the stress will have the form

$$\sigma = \sigma_{\text{мд}} - \sum_{j=1}^{n-1} C_j R_j^{j+1}(\sigma), \tag{2.30}$$

where

$$C_j = \frac{\sum_{i=1}^{j} \alpha_i h_i}{\sum_{1}^{n} \frac{h_i}{E_i}}.$$

The temperature is expressed by the formulas

$$T_n(x) = T_n^{\text{мд}}(x),$$

$$T_i(x) = T_i^{\text{мд}}(x) + q\sum_{j=i}^{n-1} R_j^{j+1}(\sigma), \quad i = 1, \ldots, n-1.$$

In this case, the solution of (2.30) always exists and is unique, and the compressive stress increases with respect to the modulus as compared with $\sigma_{\text{мд}}$.

If a steady temperature mode is given by the temperature at the hot point of the system T_o and by the q^{∞} flux, then the equation for the stress will have the form

$$\sigma = \sigma_{\text{мд}} + \sum_{j=1}^{n-1} C_j R_j^{j+1}(\sigma), \tag{2.31}$$

where

$$C_j = \frac{\sum_{i=j+1}^{n'} \alpha_i h_i}{\sum_{i=1}^{n} \frac{h_i}{E_i}} q.$$

The solution of (2.31), similar to that stated above for the case D<0 (Figures 3,4), depending on the value of T_0, q^∞ and the form of the curves $R_j^{j+1}(\sigma)$, cannot exist in the region of compressive stresses, and cannot be unique. If the solution in the region of compressive stresses exists, then $|\sigma| < |\sigma_{ид}|$ /56

3. Thermal stresses in a two-layer cylinder. Let us consider the axisymmetric thermoelastic state of two cylinders, the smaller of which $a = r_0 \leqslant r \leqslant r_1 = b$ is enclosed in the larger $r_1 = b \leqslant r \leqslant r_2 = c$, in a steady state for a given amount of heat introduced Q.

The system of equations in the case of a plane stress state has the form

$$\frac{d^2 u_i(r)}{dr^2} + \frac{1}{r}\frac{du_i(r)}{dr} - \frac{1}{r^2} u_i = \alpha_i (1+\nu_i)\frac{dT_i}{dr}, \quad (2.32)$$

$$\sigma_{rr}(r)^i = \frac{E_i}{1-\nu_i^2}\left(\frac{du_i}{dr} + \nu_i \frac{u_i}{r} - \alpha_i (1+\nu_i) T\right), \quad (2.33)$$

$$k_i \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_i}{dr}\right) = 0. \quad (2.34)$$

under the boundary conditions

$$k_1 \frac{dT_1}{dr}\Big|_{r=r_0} = -q_0^\infty, \; q_0 > 0, \quad (2.35)$$

$$k_1 \frac{dT_1}{dr}\Big|_{r_1} = k_2 \frac{dT_2}{dr}\Big|_{r_1} = \frac{1}{R(\sigma_{rr})}(T_2 - T_1)|_{r_1},$$

$$\Sigma \rho_i c_i \int_{r_{i-1}}^{r_i} T_i(r)\, r dr = Q, \quad (2.36)$$

$$u_1(r_0) = u_a, \; u_2(r_2) = u_c, \; u_1(r_1) = u_2(r_1),$$

$$\sigma_{rr}^1(r_1) = \sigma_{rr}^2(r_1) \equiv \sigma_b. \quad (2.37)$$

Here $u_i(r)$ is the displacement in the radial direction $u_r(r)$ in the i^{th} cylinder, $\sigma_{rr}(r)^i = \sigma_{rr}(r)$ in the i^{th} cylinder, σ_b--stress σ_{rr} at the point $r = r_1$, E_i--elasticity modulus, ν_i--Poisson coefficient, α--linear expansion coefficient.

In the case of plane deformation, it is necessary to replace E, ν and α by

$$\widetilde{E} = \frac{E}{1-\nu^2}, \quad \widetilde{\nu} = \frac{\nu}{1-\nu}, \quad \widetilde{\alpha} = \alpha(1+\nu).$$

/56

The solution of equations (2.34) with boundary conditions (2.35), (2.36) has the form $T_1(r) = T_1^{нд}(r) + \Delta T_c + R(\sigma_b) q_0^{\infty} \frac{r_0}{r_1}$,

$$T_2(r) = T_2^{нд}(r) + \Delta T_c, \qquad (2.38)$$

where $T_i^{нд}(r)$ - is the solution of the (2.34) with the condition of ideal contact for the same amount of heat Q:

$$T_1^{нд}(r) = -\frac{q_0 r_0}{k_1} \ln \frac{r}{r_1} + T_c - \frac{q_0 r_0}{k_2} \ln \frac{r_1}{r_2},$$

$$T_2^{нд}(r) = -\frac{q_0 r_0}{k_2} \ln \frac{r}{r_2} + T_c.$$

Substituting (2.38) into (2.36), we obtain

$$Q = Q + \Delta T_c \sum_{i=1}^{2} \rho_i c_i \frac{r_i^2 - r_{i-1}^2}{2} + R(\sigma_b) q_0^{\infty} \frac{r_0}{r_1} \rho_1 c_1 \frac{r_1^2 - r_0^2}{2}. \qquad (2.39)$$

Let us examine the stress-deformed state of the system of the cylinders. The general solution of (2.32) has the form

$$u_i(r) = \frac{(1+\nu_i)\alpha_i}{r} \int_{r_{i-1}}^{r} T_i(r) r dr + C_1 r + \frac{C_2}{r}. \qquad (2.40)$$

Thus

$$\sigma_{rr}^i(r) = -\frac{\alpha_i E_i}{r^2} \int_{r_{i-1}}^{r_i} T_i(r) r dr + \frac{EC_1^i}{1-\nu_i} - \frac{EC_2^i}{(1+\nu_i) r^2} \qquad (2.41)$$

Now giving the boundary conditions for the first cylinder

$$u(r_0) = u_a, \quad \sigma_{rr}(r_1) = \sigma_b,$$

and for the second cylinder

$$\sigma_{rr}(r_1) = \sigma_b, \quad u(r_2) = u_c,$$

we obtain the solution (2.40), (2.41) in the form

$$u_1(r) = u_a \frac{a}{r} \cdot \frac{f_1(b, r)}{f_1(b, a)} + \sigma_b \frac{1-\nu_1^2}{rE_1} \cdot \frac{b^2(r^2-a^2)}{f_1(b, a)} +$$

$$+ \frac{(1+\nu_1)\alpha_1}{r} \int_a^r Trdr + \frac{\alpha_1(1-\nu_1^2)(r^2-a^2)}{rf_1(b, a)} \int_a^b Trdr,$$

$$\sigma_{rr}^1(r) = u_a \frac{E_1 a(r^2-b^2)}{r^2 f_1(b, a)} + \sigma_b \frac{b^2 f_1(r, a)}{r^2 f_1(b, a)} -$$

$$- \frac{\alpha_1 E_1}{r^2} \int_a^r Trdr + \frac{\alpha_1 E_1}{r^2} \cdot \frac{f_1(r, a)}{f_1(b, a)} \int_a^b Trdr, \qquad /58$$

$$u_2(r) = u_c \frac{cf_2(b, r)}{rf_2(b, c)} + \sigma_b \frac{1-\nu_2^2}{E_2} \cdot \frac{b^2(r^2-c^2)}{rf_2(b, c)} +$$

$$+ \frac{(1+\nu_2)\alpha_2}{r} \int_b^r Trdr - \frac{(1+\nu_2)\alpha_2}{r} \cdot \frac{f_2(b, r)}{f_2(b, c)} \int_b^c Trdr,$$

$$\sigma_{rr}(r)^{(2)} = u_c \frac{[E_2 c(r^2-b^2)}{f_2(b, c)} + \sigma_b \frac{b^2 f_2(r, c)}{rf_2(b, c)} -$$

$$- \frac{\alpha_2 E_2}{r^2} \int_b^r Trdr - \frac{(1+\nu_2)\alpha_2 E_2(r^2-b^2)}{r^2 f_2(b, c)} \int_b^c Trdr,$$

where we introduce the notation

$$f_i(x, y) = (1+\nu_i)x^2 + (1-\nu_i)y^2.$$

Equating the displacements at the point r_1=b, we obtain the equation for determining $\sigma_{нд}$ for a given temperature distribution $T_{нд}$ and the equation connecting σ_b and ΔT_c for the temperature distribution according to the formulas (2.38)

$$\sigma_b = \sigma_{нд}(r_1) - \Delta T_c \frac{\sum_1^2 h_i \widetilde{\alpha}_i}{\sum_1^2 \frac{h_i}{E_i}} - R(\sigma_b) q_0^\infty \frac{a}{b} \cdot \frac{h_1 \widetilde{\alpha}_1}{\sum_1^2 \frac{h_i}{E_i}},$$

where

$$\widetilde{\alpha}_i = \alpha_i/(1-\nu_i),$$

$$h_1 = \frac{b(1-\nu_1^2)(b^2-a^2)}{f_1(b, a)}, \quad h_2 = \frac{b(1-\nu_2^2)(c^2-b^2)}{f_2(b, c)}.$$

ORIGINAL PAGE IS
OF POOR QUALITY

Expressing ΔT_c from equation (2.39), we obtain the equation for σ_b:

$$\sigma_b = \sigma_b^{нд} + R(\sigma_b) D, \tag{2.42}$$

where

$$D = q_0^{\infty} \frac{a}{b} \cdot \frac{h_2 \tilde{\alpha}_2 \rho_1 c_1 (b^2 - a^2) - h_1 \tilde{\alpha}_1 \rho_2 c_2 (c^2 - b^2)}{\sum \frac{h_i}{E_i} \sum \rho_i c_i (r_i^2 - r_{i-1}^2)}. \tag{2.43}$$

/59

Similarly to what was said above, the existence and uniqueness of the solution of (2.42) depends on the sign and the value of the coefficient D determined by formula (2.43), and the sign of the coefficient D now depends not only on $\frac{\alpha_2 \rho_1 c_1}{\alpha_1 \rho_2 c_2}$ but also on the geometric and elastic properties of the cylinders.

In particular, for the case of two cylinders made of identical materials

$$D = -\frac{(b^2 - a^2)(c^2 - b^2)}{2b^2 (c^2 - a^2)} \alpha E q_0^{\infty} \frac{a}{b}. \tag{2.44}$$

Equation (2.42) with the coefficient D with respect to formula (2.44) shows that the compressive stress at the site where two cylinders r=b are connected will be greater in the case of non-ideal contact. The stresses over the cylinder thickness are expressed by formulas

$$\sigma_{rr}(r)^{(1)} = \sigma_{rr}^{нд}(r)^{(1)} - \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(c^2 - b^2)(r^2 - a^2)}{2r^2 (c^2 - a^2)},$$
$$\sigma_{rr}(r)^{(2)} = \sigma_{rr}^{нд}(r)^{(2)} - \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(b^2 - a^2)(c^2 - r^2)}{2r^2 (c^2 - a^2)}. \tag{2.45}$$

Equations (50) show that for all r

$$|\sigma_{rr}| \geqslant |\sigma_{rr}^{нд}|.$$

The equation is satisfied only on the external boundary of the large cylinder r=c and on the internal boundary of the small cylinder $r = a$. Let us compare the stress $\sigma_{\theta\theta}$ and τ_{max} with the corresponding stresses in the case of ideal contact:

$$\sigma_{\theta\theta}^{(1)} = \sigma_{\theta\theta}^{\text{нд}}(r)^{(1)} - \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(c^2 - b^2)(r^2 + a^2)}{(c^2 - a^2) 2r^2},$$

$$\sigma_{\theta\theta}^{(2)} = \sigma_{\theta\theta}^{\text{нд}(2)} + \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(b^2 - a^2)(r^2 + c^2)}{(c^2 - a^2) 2r^2},$$

$$\tau_{\max}^{(1)} = \frac{\sigma_{\theta\theta}^{(1)} - \sigma_{rr}^{(1)}}{2} = \tau_{\max}^{\text{нд}(1)} - \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(c^2 - b^2) a^2}{(c^2 - a^2) 2r^2},$$

$$\tau_{\max}^{(2)} = \tau_{\max}^{\text{нд}(2)} + \alpha E q_0^{\infty} \frac{a}{b} R(\sigma_b) \frac{(b^2 - a^2) c^2}{(c^2 - a^2) 2r^2}.$$

Thus, in the case of deformation of a two-layer cylinder from one material, the non-ideal nature of the contact changes the picture of the stresses within each layer.

If the cylinders are heated within, then in our formulas $q>0$, and coefficient D in (2.42) is negative. The solution of equation (2.42) always exists and is unique. In the opposite case, i.e., if $q<0$, the existence and uniqueness of the solution depend on the intensity of the thermal flux q and the nature of the dependence $R(\sigma)$ (Figures 3,4).

REFERENCES

1. Boli, B., Winer, J. Teoriya temperaturnykh napryazheniy (Theory of temperature stresses). Moscow, Mir Press, 1964.

2. Kilikovskaya, O.A. Influence of heat resistance depending on contact pressure on stress in connected bodies. Iav. AN SSSR, MTT, No. 4, 1972.